

# Influence of a small environment on characteristic quantum features of a two-level atom coupled to a cavity field

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## Abstract

In this paper we investigate some aspects of the dynamics of a quantum system constituted by a two-level atom (atom 1) in interaction with a single mode cavity field. The field is considered to be weakly coupled to a third subsystem: a second two-level atom (atom 2), in a mixed state, which will play the role of an external environment. We analyze the influence of such a small environment on the dynamics of quantities such as the linear entropy (state purity) and the atomic dipole squeezing of atom 1, as well as the entanglement between atom 1 and the field. In order to study a general situation, we obtain the full analytical solution of the two atom Tavis-Cummings model in which the constituent atoms may be coupled with different strengths to the field and also have different frequency detunings. We show that nonclassical features associated to the main system may be significantly degraded by the action of the small environment, if atom 2 is resonantly coupled to the field. We also demonstrate that the nonclassical behaviour of the system may be restored if we detune the field from the transition frequency of atom 2, the environment.

## 1 Introduction

The evolution of quantum systems in contact with external environments has been the subject of investigation for quite some time [1]. The environment is usually modeled as a (quantum) system having a large number of degrees of freedom, viz., a reservoir, and number of methods, mainly perturbative, have been developed in order to study the influence of the surroundings on the behaviour of quantum systems [1]. It is in general possible to derive an equation describing the evolution of the reduced density operator of the system of interest - the master equation, obtained by tracing over the degrees of freedom of the environment. Generally speaking, the coupling to an external environment has a detrimental action on the quantum properties of the system of interest, causing effects such as the loss of quantum coherence (decoherence) [2, 3, 4]. The models of environment mostly rely on the assumption of the existence of an ideal (large) reservoir, which naturally leads to irreversibility and relaxation features. Consider a single mode (cavity) field of the quantized field as being the system of interest. Cavity losses (dissipation) may be modeled via a reservoir constituted by a large collection of either independent field modes [1] or a beam of two-level atoms [5]. Both procedures lead basically to the same master equation for reduced density operator of the cavity field. Nevertheless, environments with other types of structure, not necessarily having a large number of degrees of freedom, may also occur in nature. Thus, one may think in a situation in which a system of interest is coupled to an uncontrollable surrounding environment having a small number of degrees of freedom. We could cite the interesting experiment (double photoionization of H<sub>2</sub> molecules) presented in reference [6], where it is shown that even a single electron, constituting a “minimal environment” may strongly affect the interference fringes relative to a second electron (system of interest). The effects of small environments constituted by either a single harmonic oscillator or a single spin-1/2 particle have also been addressed in [7, 8] and [9]. As a matter of fact, one of the smallest possible environments could be constituted by a single two-level system, opposed to the model of a reservoir containing a large number of atoms [5]. In [10] it is studied a system made of a two-level system (atom 1) coupled to an oscillator (field mode) which is itself in interaction with a minimal environment constituted by a second two-level system (atom 2). This is formally the well-known two-atom Tavis-Cummings model [11], but where an asymmetric partition of the system has been considered; the system of interest being constituted by atom 1 + field, while a partial trace is performed over the environment (atom 2). In [10], the discussion is restricted to the exact resonance case, i.e., the atom 1(2)-field detunings being equal ( $\Delta_1 = \Delta_2 = 0$ ). Nevertheless the atoms are assumed to be coupled with different strengths to the field ( $\lambda_1 \neq \lambda_2$ ); while atom 1 and the field are assumed to be strongly coupled, the field is weakly coupled to its environment. If the field is completely isolated from atom 2, i.e.,  $\lambda_2 = 0.0$ , we have the

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basic model of quantum optical resonance known as the Jaynes-Cummings model [12], a particular case of the Tavis-Cummings model for  $N = 1$ . The Jaynes-Cummings model has well known features; for instance, coherent Rabi oscillations of the atomic inversion as well as the linear entropy of the atom,  $S = 1 - \text{Tr}(\rho_a^2)$ ,<sup>2</sup> if the field is initially prepared in a Fock state. Such a characteristic behavior may be useful to evaluate the interference of external systems on the regular evolution of a quantum system, and thus we could focus our attention on the reduced density operator of atom 1,  $\rho_{a1}$ , obtained from a partial trace over both the variables of atom 2 and the field. One of the conclusions of [10] is that the evolution of the linear entropy strongly depends on the degree of mixedness of atom 2, meaning that even a small (although noisy) environment may significantly disturb the evolution of two coupled quantum systems (atom 1 + field). Here we would like to further investigate in which way the quantum features of the system of interest are affected due to its coupling to such a small environment. We analyze the evolution of non-classical effects like the atomic dipole squeezing [13, 14], as well the entanglement between atom 1 and the field. In general non-classical effects are susceptible to external influences, and we would like to find to what extent an unwanted small environment with few degrees of freedom may affect the squeezing and entanglement properties of our system. We also address a more general situation, in which the detunings  $\Delta_1, \Delta_2$  as well as the coupling strengths  $\lambda_1, \lambda_2$  are different. This provides us additional flexibility. For instance, we would like to minimize the destructive effect of the environment, and one possible way of doing it is by controlling the detuning between the field and the environment itself. We note that the analytical solutions of the two-atom Tavis-Cummings model found in the literature are restricted to particular cases. Namely, the atom-field coupling constants may be assumed to be equal (identical atoms) [15], or different (non-identical atoms), but still having the frequency of the field equal to the atomic transition frequencies [16, 14]. A step further is given in [17], where it is presented a solution of the model for non-identical atoms but being both of them either on resonance or out of resonance with the field. Here we work out an exact analytical solution of the two-atom Tavis-Cummings model for distinct coupling constants,  $\lambda_1, \lambda_2$  and arbitrary detunings,  $\Delta_1, \Delta_2$ . We remark that the solution in this general case has a considerably more complicated structure than the solution for  $\Delta_1 = \Delta_2$  (equal detunings).

Our paper is organized as follows. In section 2 we obtain the general analytical solution of the two-atom Tavis-Cummings model with arbitrary detunings and couplings. In section 3 we discuss the time evolution of the linear entropy, the atomic dipole squeezing of atom 1 as well as the atom 1-field entanglement, considering atom 2 as a disturbance (environment). We also show how it is possible to restore quantum coherence, the atomic dipole squeezing and entanglement by controlling the frequency detuning between the field and the environment (atom 2). In section 4 we summarize our conclusions.

## 2 Tavis-Cummings model: an analytical solution for different coupling constants and different detunings

The two-atom Tavis-Cummings model is described by the following Hamiltonian (under the rotating wave approximation and making  $\hbar = 1$ )

$$H = \frac{\omega_1}{2}\sigma_1^z + \frac{\omega_2}{2}\sigma_2^z + \omega a^\dagger a + \lambda_1(a\sigma_1^+ + a^\dagger\sigma_1^-) + \lambda_2(a\sigma_2^+ + a^\dagger\sigma_2^-), \quad (1)$$

where  $a$  ( $a^\dagger$ ) are the annihilation (creation) operators associated to the field mode, with frequency  $\omega$ ;  $\sigma_i^z, \sigma_i^+, \sigma_i^-$  are the de Pauli operators relative to the “i-th” atom, each atom having transition frequency  $\omega_i$ . We may rewrite the Hamiltonian above in terms of the detunings  $\Delta_1 = \omega_1 - \omega$  and  $\Delta_2 = \omega_2 - \omega$ , as

$$H = \omega \left( \frac{\sigma_1^z + \sigma_2^z}{2} + a^\dagger a \right) + \frac{\Delta_1}{2}\sigma_1^z + \frac{\Delta_2}{2}\sigma_2^z + \lambda_1(a\sigma_1^+ + a^\dagger\sigma_1^-) + \lambda_2(a\sigma_2^+ + a^\dagger\sigma_2^-). \quad (2)$$

The Hamiltonian (2) may be split in two parts

$$H = H_0 + H_1, \quad (3)$$

where

$$H_0 = \omega \left( \frac{\sigma_1^z + \sigma_2^z}{2} + a^\dagger a \right) \quad (4)$$

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<sup>2</sup>Being  $\rho_a$  the reduced atomic density operator.

represents a conserved quantity, and

$$H_1 = \frac{\Delta_1}{2} \sigma_1^z + \frac{\Delta_2}{2} \sigma_2^z + \lambda_1 (a \sigma_1^+ + a^\dagger \sigma_1^-) + \lambda_2 (a \sigma_2^+ + a^\dagger \sigma_2^-) \quad (5)$$

is the interaction part.

The Schrödinger equation in the interaction representation reads

$$i \frac{d}{dt} |\psi_I(t)\rangle = H_I |\psi_I(t)\rangle, \quad (6)$$

with  $H_I = \exp(i H_0 t) H_1 \exp(-i H_0 t)$ , and  $H_I = H_1$ .

The Hamiltonian  $H_I$  induces transitions between the states  $|e_1, e_2, n\rangle$ ,  $|e_1, g_2, n+1\rangle$ ,  $|g_1, e_2, n+1\rangle$ ,  $|g_1, g_2, n+2\rangle$ , and therefore we may write the following *ansatz*

$$\begin{aligned} |\psi_I(t)\rangle &= C_{1,n}(t) |e_1, e_2, n\rangle + C_{2,n}(t) |e_1, g_2, n+1\rangle \\ &+ C_{3,n}(t) |g_1, e_2, n+1\rangle + C_{4,n}(t) |g_1, g_2, n+2\rangle. \end{aligned} \quad (7)$$

After substituting the proposed solution (7) above in the Schrödinger equation, we obtain the corresponding set of coupled differential equations for the amplitudes  $C_{i,n}(t)$

$$\begin{aligned} i \dot{C}_{1,n} &= \left( \frac{\Delta_1 + \Delta_2}{2} \right) C_{1,n} + \lambda_2 \sqrt{n+1} C_{2,n} + \lambda_1 \sqrt{n+1} C_{3,n}, \\ i \dot{C}_{2,n} &= \lambda_2 \sqrt{n+1} C_{1,n} + \left( \frac{\Delta_1 - \Delta_2}{2} \right) C_{2,n} + \lambda_1 \sqrt{n+2} C_{4,n}, \\ i \dot{C}_{3,n} &= \lambda_1 \sqrt{n+1} C_{1,n} + \left( \frac{\Delta_2 - \Delta_1}{2} \right) C_{3,n} + \lambda_2 \sqrt{n+2} C_{4,n}, \\ i \dot{C}_{4,n} &= \lambda_1 \sqrt{n+2} C_{2,n} + \lambda_2 \sqrt{n+2} C_{3,n} - \left( \frac{\Delta_1 + \Delta_2}{2} \right) C_{4,n}. \end{aligned} \quad (8)$$

Because the exact analytical solutions of the equations (8) are rather lengthy, we have placed them in the appendix.

### 3 Numerical results: quantum state purity, dipole squeezing and entanglement

Now we would like to discuss the reduced dynamics of the quantum system, basically focusing on the properties associated to atom 1. In order to do so, we should first trace, the total density operator  $\rho(t)$ , over the variables of atom 2 (the “environment”), obtaining the joint atom 1-field density operator, or  $\rho(t)_{a1,f} = \text{Tr}_{a2} \rho(t)$ . For a completely isolated atom (Jaynes Cummings model, [12]), we know that the linear entropy has a completely reversible behaviour, provided the field is initially prepared in a Fock state. Moreover, non-classical features such as squeezing [13] may also arise during the atom-field interaction. If we want to focus solely on the atomic properties, we should perform a further partial trace over the field variables, i.e., calculate the reduced density operator relative to atom 1,  $\rho(t)_{a1} = \text{Tr}_f [\rho(t)_{a1,f}]$ . Here we are going to assume initial conditions of the form:  $\rho(0) = \rho_{a1}(0) \otimes \rho_f(0) \otimes \rho_{a2}(0)$ , with

$$\rho_{a1}(0) = |\phi_1\rangle \langle \phi_1|, \quad \rho_f(0) = |N\rangle \langle N|, \quad \rho_{a2}(0) = p |e_2\rangle \langle e_2| + (1-p) |g_2\rangle \langle g_2|. \quad (9)$$

In other words, having atom 1 prepared in the state  $|\phi_1\rangle = \cos(\frac{\theta}{2}) |g_1\rangle + \sin(\frac{\theta}{2}) e^{i\phi} |e_1\rangle$  and the field prepared in a Fock state  $|N\rangle$ , with  $N$  being either  $N=1$  or  $N=0$ . Given that atom 2 is treated here as a noisy “environment” perturbing the atom 1-field system, we consider it initially in a statistical mixture of its ground and excited states,  $\rho_{a2}(0)$ ; here we are going to address the case of a maximally mixed environment, or  $p=1/2$ . We may now discuss some of the quantum dynamical features of the system.

### 3.1 Reduced dynamics of atom 1: linear entropy evolution

The linear entropy is used to quantify the degree of purity of a quantum state. In the case of atom 1, it may be written as

$$S(t) = 1 - \text{Tr}_{a1}(\rho_{a1}^2) = 2 \left[ \alpha - \alpha^2 - |\gamma|^2 \right], \quad (10)$$

where

$$\begin{aligned} \alpha(t) = & \sin^2\left(\frac{\theta}{2}\right) \left\{ p \left[ |C_{3,1}^{(I)}|^2 + |C_{4,1}^{(I)}|^2 \right] + (1-p) \left[ |C_{3,0}^{(II)}|^2 + |C_{4,0}^{(II)}|^2 \right] \right\} \\ & + \cos^2\left(\frac{\theta}{2}\right) \left\{ p \left[ |C_{3,0}^{(III)}|^2 + |C_{4,0}^{(III)}|^2 \right] + (1-p) \left[ |C_{3,-1}^{(IV)}|^2 + |C_{4,-1}^{(IV)}|^2 \right] \right\}, \end{aligned} \quad (11)$$

and

$$\begin{aligned} \gamma(t) = & e^{-i\phi} \frac{\sin(\theta)}{2} \left\{ p \left[ C_{1,1}^{(I)*} C_{3,0}^{(III)} + C_{2,1}^{(I)*} C_{4,0}^{(III)} \right] + \right. \\ & \left. + (1-p) \left[ C_{1,0}^{(II)*} C_{3,-1}^{(IV)} + C_{2,0}^{(II)*} C_{4,-1}^{(IV)} \right] \right\} \end{aligned} \quad (12)$$

are the populations and coherences (respectively) of the reduced density operator of atom 1

$$\rho_{a1}(t) = \alpha |g_1\rangle \langle g_1| + (1-\alpha) |e_1\rangle \langle e_1| + \gamma |g_1\rangle \langle e_1| + \gamma^* |e_1\rangle \langle g_1|. \quad (13)$$

Here the superscript  $i$ , ( $i = I, II, III, IV$ ) in the amplitudes  $C_{j,n}^{(i)}$  in equations (11) and (12) above are related to the initial conditions. The amplitudes  $C_{j,n}(t)$  may be written as a linear combination of the initial conditions as  $C_{j,n}(t) = \sum_{j,m=1}^4 A_{j,m} c_j(0)$ . The notation employed is such that  $C_{1,1}^{(I)}$ , for instance, indicates that  $C_1(0) = 1$  and the remaining coefficients are zero; the superscript  $(II)$  in  $C_{2,0}^{(II)}$  indicates that  $C_2(0) = 1$ , and the remaining coefficients are zero and so on.

We would like now to analyze the effect of  $\Delta_2$  (atom 2-field detuning) on the time evolution of the linear entropy of atom 1. In figure (1) we have plots of the linear entropy of atom 1 as a function of time considering different values of  $\Delta_2$ . The field is initially prepared in the one photon Fock state  $|1\rangle$ , atom 1 in its excited state  $|e_1\rangle$ , and atom 2 in a maximally mixed state,  $\rho_{a2}(0) = \frac{1}{2}(|g_2\rangle \langle g_2| + |e_2\rangle \langle e_2|)$ .

As seen in figure (1a), the linear entropy is a periodic function of time (Rabi oscillations) if atom 2 is decoupled from the field, i.e., for  $\lambda_2 = 0.0$ . If the interaction between atom 2 and the field is turned on (f  $\lambda_2 = 0.1$ ), the evolution of the linear entropy becomes very irregular, as shown in figure (1b), which characterizes a destructive effect due to an unwanted coupling to a noisy sub-system. Nevertheless, it is possible to restore periodicity by controlling the atom 2-field frequency detuning. If  $\Delta_2$  is increased, we expect less influence of atom 2 over the dynamics of the system of interest. In fact, as shown in figure (1c) and figure (1d), periodicity may be re-established for a sufficiently large  $\Delta_2$ . We remark that atom 1 is kept on resonance with the field ( $\Delta_1 = 0.0$ ) as well as strongly coupled to it ( $\lambda_1 = 1.0$ ).

### 3.2 Reduced dynamics of atom 1: atomic dipole squeezing

Atomic dipole squeezing is a non-classical effect that may arise in the Jaynes-Cummings model [13]. An atom is said to be dipole squeezed if the quantum fluctuations of the atomic dipole are below the fundamental limit imposed by the Heisenberg inequalities. The components of the (slowly varying) atomic dipole operator (for atom 1) may be written as [13]

$$\begin{aligned} \sigma_x &= \sigma_1^+ e^{i\omega_1 t} + \sigma_1^- e^{-i\omega_1 t}, \\ \sigma_y &= \frac{1}{i} (\sigma_1^+ e^{i\omega_1 t} - \sigma_1^- e^{-i\omega_1 t}). \end{aligned} \quad (14)$$

The operators above do not commute ( $[\sigma_x, \sigma_y] = 2i\sigma_z$ ), and thus they should obey the Heisenberg inequality  $(\Delta\sigma_x)(\Delta\sigma_y) \geq |\langle\sigma_z\rangle|$ . Atomic dipole squeezing is verified if

$$(\Delta\sigma_{x,y})^2 < |\langle\sigma_z\rangle|, \quad (15)$$

where  $(\Delta\sigma_x)^2 = 1 - 4 \left( \text{Re} \langle \sigma_1^- \rangle e^{-i\omega_1 t} \right)^2$  and  $(\Delta\sigma_y)^2 = 1 - 4 \left( \text{Im} \langle \sigma_1^- \rangle e^{-i\omega_1 t} \right)^2$ . The conditions for squeezing may be written in terms of the functions  $s_1$  and  $s_2$  (indexes of squeezing) as

$$s_1 = \frac{1 - 4 \left( \text{Re} \langle \sigma_1^- \rangle e^{-i\omega_1 t} \right)^2}{|\langle \sigma_z \rangle|} < 1 \quad \text{or} \quad s_2 = \frac{1 - 4 \left( \text{Im} \langle \sigma_1^- \rangle e^{-i\omega_1 t} \right)^2}{|\langle \sigma_z \rangle|} < 1, \quad (16)$$

where  $\omega_1$  is the frequency of atom 1, and  $\langle \sigma_z \rangle$  is its corresponding atomic inversion. Here  $\langle \sigma_\alpha \rangle = \text{Tr}_1(\sigma_\alpha \rho_{a1})$ .

We may rewrite the indexes of squeezing,  $s_1$  and  $s_2$ , in terms of the functions  $\alpha(t)$  and  $\gamma(t)$  above, or

$$\begin{aligned} s_1 &= \frac{1 - 4 \left( \text{Re}[\gamma] \cos(\omega_1 t) - \text{Im}[\gamma] \sin(\omega_1 t) \right)^2}{|1 - 2\alpha(t)|} < 1, \\ s_2 &= \frac{1 - 4 \left( \text{Re}[\gamma] \sin(\omega_1 t) + \text{Im}[\gamma] \cos(\omega_1 t) \right)^2}{|1 - 2\alpha(t)|} < 1. \end{aligned} \quad (17)$$

Now we choose specific initial conditions which allow dipole squeezing in the case of having atom 2 completely decoupled from the field ( $\lambda_2 = 0.0$ ), which corresponds to the Jaynes-Cummings model. Atom 1 is assumed to be prepared in the superposition state  $|\phi_1(0)\rangle = \cos(0.6)|g_1\rangle + \sin(0.6)|e_1\rangle$ ; atom 2 in the maximally mixed state,  $\rho_{a2}(0) = \frac{1}{2}(|g_2\rangle\langle g_2| + |e_2\rangle\langle e_2|)$  and the field in the one photon Fock state,  $|1\rangle$ . In figure (2) we have plotted the dipole squeezing index  $s_1$  relative to atom 1 as a function of time. We note that in the absence of the “environment” (atom 2), dipole squeezing occurs for a few narrow intervals of time [figure (2a)]. However, if atom 2 is weakly coupled to the field, e.g.,  $\lambda_2 = 0.1$ , the dipole squeezing is inhibited due to the action of the environment [see figure (2b)]. Nevertheless, similarly to what we have seen in the previous subsection, dipole squeezing may be restored for a large enough detuning between atom 2 and the field, as shown in figures (2c) and (2d).

### 3.3 Atom 1-field entanglement

We may quantify the entanglement between atom 1 and the field by evaluating the negativity [18], an entanglement measure especially convenient in this case. Now we assume the following initial conditions: the field initially prepared in the vacuum state  $|0\rangle$ , atom 1 in its excited state  $|e_1\rangle$ , and atom 2 in a maximally mixed state,  $\rho_{a2}(0) = \frac{1}{2}(|g_2\rangle\langle g_2| + |e_2\rangle\langle e_2|)$ . The negativity may be calculated from the time-dependent atom 1-field reduced density operator

$$\begin{aligned} \rho_{a1-f}(t) &= \rho_{11}|g_1, 0\rangle\langle g_1, 0| + \rho_{22}|g_1, 1\rangle\langle g_1, 1| + \rho_{33}|g_1, 2\rangle\langle g_1, 2| + \\ &\quad + \rho_{44}|e_1, 0\rangle\langle e_1, 0| + \rho_{55}|e_1, 1\rangle\langle e_1, 1| + \rho_{24}|g_1, 1\rangle\langle e_1, 0| + \\ &\quad + \rho_{24}^*|e_1, 0\rangle\langle g_1, 1| + \rho_{35}|g_1, 2\rangle\langle e_1, 1| + \rho_{35}^*|e_1, 1\rangle\langle g_1, 2|. \end{aligned}$$

The relevant matrix elements are

$$\rho_{11} = \frac{1}{2} |C_{3,-1}^{(II)}|^2, \quad \rho_{22} = \frac{1}{2} \left( |C_{3,0}^{(I)}|^2 + |C_{4,-1}^{(II)}|^2 \right),$$

$$\rho_{33} = \frac{1}{2} |C_{4,0}^{(I)}|^2, \quad \rho_{44} = \frac{1}{2} \left( |C_{1,0}^{(I)}|^2 + |C_{2,-1}^{(II)}|^2 \right),$$

$$\rho_{55} = \frac{1}{2} |C_{2,0}^{(I)}|^2, \quad \rho_{35} = \frac{1}{2} \left( C_{2,0}^{(I)} \right)^* C_{4,0}^{(I)},$$

$$\rho_{24} = \frac{1}{2} \left[ \left( C_{1,0}^{(I)} \right)^* C_{3,0}^{(I)} + \left( C_{2,-1}^{(II)} \right)^* C_{4,-1}^{(II)} \right].$$

Thus the negativity  $\mathcal{N}$  may be expressed as

$$\mathcal{N} = 2 \sum_{i=1}^2 |a_i|$$

$$a_1 = \frac{1}{2} \left( \rho_{22} - \sqrt{\rho_{22}^2 + 4|\rho_{35}|^2} \right), \quad a_2 = \frac{1}{2} \left( \rho_{11} + \rho_{55} - \sqrt{(\rho_{11} - \rho_{55})^2 + 4|\rho_{24}|^2} \right).$$

We have inserted a factor of 2 so that the maximum of entanglement corresponds to 1. We note that although the negativity is somehow related to the linear entropy, they may not have the same behaviour, given that in this case the reduced atom 1-field density operator is obtained from a mixed state. In figure (3) we have plotted the negativity  $\mathcal{N}$  as a function of time. For the initial conditions chosen here, the system has a simple evolution, i.e., a oscillatory behaviour as shown in figure (3a), provided the system is isolated (decoupled environment). However, if atom 2 (environment) is resonantly coupled to the field, the entanglement is severely damped [see figure (3b)], similarly to what happens to the linear entropy and atomic dipole squeezing. Again, entanglement may be restored if  $\Delta_2$  is increased, as shown in figures (3c) and (3d).

## 4 Conclusions

We have presented a study of the dynamics of a bipartite quantum system (atom 1-field) in which the field is weakly coupled ( $\lambda_2 \approx \lambda_1/10$ ) to a small environment (atom 2). A single atom corresponds to the smallest possible environment considering the atomic beam model for a reservoir [5]. In the many atoms model a Markovian master equation is obtained via a perturbative method, which leads to irreversible field damping and decoherence [1, 5]. However, in the “single atom environment” model an exact, non-perturbative solution is allowed, and as found in [10] (restricted to the resonant case), the linear entropy of the quantum system undergoes an irreversible-like evolution which is very sensitive to the mixedness of the small environment. Here we have considered a more general situation, by analytically solving the two-atom Tavis-Cummings model for non-identical atoms, with different coupling constants, ( $\lambda_1 \neq \lambda_2$ ) and different detunings ( $\Delta_1 \neq \Delta_2$ ). Besides, we have also investigated the evolution of important quantities of the system related to non-classical behaviour, such as the dipole squeezing of atom 1 and the negativity, which quantifies the atom 1-field quantum entanglement. The small environment (atom 2), has been assumed to be initially in a maximally mixed state. Naturally in this simple model, contrarily to what happens in the case of a many atom reservoir, there is no irreversible loss of coherence. Instead, due to the incommensurate frequencies characteristic of the model, the resulting dynamics is quasi-periodic, and we should expect quasi-recurrences of the considered physical quantities at longer time-scales. Nevertheless, due to the incoming “thermal noise” from the environment, the atomic dipole squeezing as well as the atom 1-field entanglement are considerably degraded (in the resonant case,  $\Delta_2 = 0.0$ ), specially if one is restricted to shorter time scales (up to a few Rabi cycles, as shown in the figures). In particular, we have verified that dipole squeezing is completely suppressed. Squeezing only exists if the index defined in equation (17) becomes less than one [see figure (2)], and it occurs at relatively narrow time intervals.

We have also obtained a full analytical solution of the two-atom Tavis-Cummings model for non-identical atoms, which allowed us to assess the effect of the atom 2-field detuning on the dynamics of the system. Indeed there is a competition between the environment-field coupling and the environment-field detuning, but although we were able to determine explicitly the dependence of the quantum state of the system on the parameters ( $\lambda_1, \lambda_2, \Delta_1, \Delta_2$ ), the lengthy expressions hindered a more detailed analysis. Yet, it is possible to have some degree of control over the dynamics of the system and circumvent the destructive effect of the environment. By increasing the atom 2-field detuning  $\Delta_2$ , there will be an effective decoupling of the small environment, and we expect a restoration of the non-classical properties such as squeezing and entanglement. For instance, we have found that it is sufficient to have a (environment-field) detuning around  $\Delta_2 = 1.0$  in order to recover the atomic dipole squeezing property of atom 1. However, larger values of detuning would be required to restore the atom-field entanglement, as it is clearly seen in figure (3)]. We shall also point out that, as discussed in [10], any additional disturbances combined to the action of the small environment will turn out to be very detrimental for quantum coherence.

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## Appendix. Solution of the two-atom Tavis-Cummings model

Here we present the full analytical solution of the two-atom Tavis-Cummings model. In order to solve the system of coupled differential equations (8), we have employed the Laplace transform method, and the set of differential equations is transformed to the following set of algebraic equations

$$\begin{aligned} \left(-\frac{\Delta_1 + \Delta_2}{2} + i s\right) \tilde{C}_1 - \lambda_2 \sqrt{n+1} \tilde{C}_2 - \lambda_1 \sqrt{n+1} \tilde{C}_3 &= i c_1(0) \\ -\lambda_1 \sqrt{n+1} \tilde{C}_1 + \left(\frac{\Delta_2 - \Delta_1}{2} + i s\right) \tilde{C}_2 - \lambda_1 \sqrt{n+2} \tilde{C}_4 &= i c_2(0) \\ -\lambda_1 \sqrt{n+1} \tilde{C}_1 + \left(\frac{\Delta_1 - \Delta_2}{2} + i s\right) \tilde{C}_3 - \lambda_2 \sqrt{n+2} \tilde{C}_4 &= i c_3(0) \\ -\lambda_1 \sqrt{n+2} \tilde{C}_2 - \lambda_2 \sqrt{n+2} \tilde{C}_3 + \left(\frac{\Delta_1 + \Delta_2}{2} + i s\right) \tilde{C}_4 &= i c_4(0). \end{aligned}$$

For simplicity we have denoted  $\tilde{C}_j(s) = \tilde{C}_j$  in the equations above. The subsequent steps involve the solution of polynomials up to fourth degree, and after some involved calculations we obtain the full solution.

The diagonal terms are

$$\begin{aligned} A_{11} = \frac{1}{54} & \left\{ \frac{e^{-\frac{1}{2}t(\sqrt{X}-\sqrt{Y_1})} \left( 18(\sqrt{3}+i) D_1 J_1 \left( -2ik_1 + 3\sqrt{X} - 3\sqrt{Y_1} \right) - 432D_1^3 - 4iJ_1^3 + \left( 2ik_1 - 3\sqrt{X} + 3\sqrt{Y_1} \right)^3 \right)}{\sqrt{Y_1} \left( -4\sqrt{X}\sqrt{Y_1} + 4X + Y_1 - Y_2 \right)} \right. \\ & + \frac{e^{-\frac{1}{2}t(\sqrt{X}+\sqrt{Y_1})} \left( 18(\sqrt{3}+i) iD_1 J_1 \left( 2k_1 + 3i \left( \sqrt{X} + \sqrt{Y_1} \right) \right) + 432D_1^3 + i \left( 4J_1^3 + \left( 2k_1 + 3i \left( \sqrt{X} + \sqrt{Y_1} \right) \right)^3 \right) \right)}{\sqrt{Y_1} \left( 4\sqrt{X}\sqrt{Y_1} + 4X + Y_1 - Y_2 \right)} \\ & + \frac{e^{\frac{1}{2}t(\sqrt{X}-\sqrt{Y_2})} \left( 18(\sqrt{3}+i) D_1 J_1 \left( 2ik_1 + 3\sqrt{X} - 3\sqrt{Y_2} \right) + 432D_1^3 + 4iJ_1^3 + \left( -2ik_1 - 3\sqrt{X} + 3\sqrt{Y_2} \right)^3 \right)}{\sqrt{Y_2} \left( -4\sqrt{X}\sqrt{Y_2} + 4X - Y_1 + Y_2 \right)} \\ & \left. + \frac{e^{\frac{1}{2}t(\sqrt{X}+\sqrt{Y_2})} \left( 18(1-i\sqrt{3}) D_1 J_1 \left( 2k_1 - 3i \left( \sqrt{X} + \sqrt{Y_2} \right) \right) - 432D_1^3 - i \left( 4J_1^3 + \left( 2k_1 - 3i \left( \sqrt{X} + \sqrt{Y_2} \right) \right)^3 \right) \right)}{\sqrt{Y_2} \left( 4\sqrt{X}\sqrt{Y_2} + 4X - Y_1 + Y_2 \right)} \right\}, \\ \\ A_{22} = \frac{1}{54} & \left\{ \frac{e^{-\frac{1}{2}t(\sqrt{X}-\sqrt{Y_1})} \left( 18(\sqrt{3}+i) D_2 J_2 \left( -2ik_2 + 3\sqrt{X} - 3\sqrt{Y_1} \right) - 432D_2^3 - 4iJ_2^3 + \left( 2ik_2 - 3\sqrt{X} + 3\sqrt{Y_1} \right)^3 \right)}{\sqrt{Y_1} \left( -4\sqrt{X}\sqrt{Y_1} + 4X + Y_1 - Y_2 \right)} \right. \\ & + \frac{e^{-\frac{1}{2}t(\sqrt{X}+\sqrt{Y_1})} \left( 18(\sqrt{3}+i) iD_2 J_2 \left( 2k_2 + 3i \left( \sqrt{X} + \sqrt{Y_1} \right) \right) + 432D_2^3 + i \left( 4J_2^3 + \left( 2k_2 + 3i \left( \sqrt{X} + \sqrt{Y_1} \right) \right)^3 \right) \right)}{\sqrt{Y_1} \left( 4\sqrt{X}\sqrt{Y_1} + 4X + Y_1 - Y_2 \right)} \\ & + \frac{e^{\frac{1}{2}t(\sqrt{X}-\sqrt{Y_2})} \left( 18(\sqrt{3}+i) D_2 J_2 \left( 2ik_2 + 3\sqrt{X} - 3\sqrt{Y_2} \right) + 432D_2^3 + 4iJ_2^3 + \left( -2ik_2 - 3\sqrt{X} + 3\sqrt{Y_2} \right)^3 \right)}{\sqrt{Y_2} \left( -4\sqrt{X}\sqrt{Y_2} + 4X - Y_1 + Y_2 \right)} \\ & \left. + \frac{e^{\frac{1}{2}t(\sqrt{X}+\sqrt{Y_2})} \left( 18(1-i\sqrt{3}) D_2 J_2 \left( 2k_2 - 3i \left( \sqrt{X} + \sqrt{Y_2} \right) \right) - 432D_2^3 - i \left( 4J_2^3 + \left( 2k_2 - 3i \left( \sqrt{X} + \sqrt{Y_2} \right) \right)^3 \right) \right)}{\sqrt{Y_2} \left( 4\sqrt{X}\sqrt{Y_2} + 4X - Y_1 + Y_2 \right)} \right\}, \end{aligned}$$

$$\begin{aligned}
A_{33} = & \frac{1}{54} \left\{ \frac{e^{-\frac{1}{2}t(\sqrt{X}-\sqrt{Y_1})} \left( 18(\sqrt{3}+i) D_3 J_3 \left( -2ik_3 + 3\sqrt{X} - 3\sqrt{Y_1} \right) - 432D_3^3 - 4iJ_3^3 + \left( 2ik_3 - 3\sqrt{X} + 3\sqrt{Y_1} \right)^3 \right)}{\sqrt{Y_1} \left( -4\sqrt{X}\sqrt{Y_1} + 4X + Y_1 - Y_2 \right)} \right. \\
& + \frac{e^{-\frac{1}{2}t(\sqrt{X}+\sqrt{Y_1})} \left( 18(\sqrt{3}+i) iD_3 J_3 \left( 2k_3 + 3i \left( \sqrt{X} + \sqrt{Y_1} \right) \right) + 432D_3^3 + i \left( 4J_3^3 + \left( 2k_3 + 3i \left( \sqrt{X} + \sqrt{Y_1} \right) \right)^3 \right) \right)}{\sqrt{Y_1} \left( 4\sqrt{X}\sqrt{Y_1} + 4X + Y_1 - Y_2 \right)} \\
& + \frac{e^{\frac{1}{2}t(\sqrt{X}-\sqrt{Y_2})} \left( 18(\sqrt{3}+i) D_3 J_3 \left( 2ik_3 + 3\sqrt{X} - 3\sqrt{Y_2} \right) + 432D_3^3 + 4iJ_3^3 + \left( -2ik_3 - 3\sqrt{X} + 3\sqrt{Y_2} \right)^3 \right)}{\sqrt{Y_2} \left( -4\sqrt{X}\sqrt{Y_2} + 4X - Y_1 + Y_2 \right)} \\
& \left. + \frac{e^{\frac{1}{2}t(\sqrt{X}+\sqrt{Y_2})} \left( 18(1-i\sqrt{3}) D_3 J_3 \left( 2k_3 - 3i \left( \sqrt{X} + \sqrt{Y_2} \right) \right) - 432D_3^3 - i \left( 4J_3^3 + \left( 2k_3 - 3i \left( \sqrt{X} + \sqrt{Y_2} \right) \right)^3 \right) \right)}{\sqrt{Y_2} \left( 4\sqrt{X}\sqrt{Y_2} + 4X - Y_1 + Y_2 \right)} \right\},
\end{aligned}$$

$$\begin{aligned}
A_{44} = & \frac{1}{54} \left\{ \frac{e^{-\frac{1}{2}t(\sqrt{X}-\sqrt{Y_1})} \left( 18(\sqrt{3}+i) D_4 J_4 \left( -2ik_4 + 3\sqrt{X} - 3\sqrt{Y_1} \right) - 432D_4^3 - 4iJ_4^3 + \left( 2ik_4 - 3\sqrt{X} + 3\sqrt{Y_1} \right)^3 \right)}{\sqrt{Y_1} \left( -4\sqrt{X}\sqrt{Y_1} + 4X + Y_1 - Y_2 \right)} \right. \\
& + \frac{e^{-\frac{1}{2}t(\sqrt{X}+\sqrt{Y_1})} \left( 18(\sqrt{3}+i) iD_4 J_4 \left( 2k_4 + 3i \left( \sqrt{X} + \sqrt{Y_1} \right) \right) + 432D_4^3 + i \left( 4J_4^3 + \left( 2k_4 + 3i \left( \sqrt{X} + \sqrt{Y_1} \right) \right)^3 \right) \right)}{\sqrt{Y_1} \left( 4\sqrt{X}\sqrt{Y_1} + 4X + Y_1 - Y_2 \right)} \\
& + \frac{e^{\frac{1}{2}t(\sqrt{X}-\sqrt{Y_2})} \left( 18(\sqrt{3}+i) D_4 J_4 \left( 2ik_4 + 3\sqrt{X} - 3\sqrt{Y_2} \right) + 432D_4^3 + 4iJ_4^3 + \left( -2ik_4 - 3\sqrt{X} + 3\sqrt{Y_2} \right)^3 \right)}{\sqrt{Y_2} \left( -4\sqrt{X}\sqrt{Y_2} + 4X - Y_1 + Y_2 \right)} \\
& \left. + \frac{e^{\frac{1}{2}t(\sqrt{X}+\sqrt{Y_2})} \left( 18(1-i\sqrt{3}) D_4 J_4 \left( 2k_4 - 3i \left( \sqrt{X} + \sqrt{Y_2} \right) \right) - 432D_4^3 - i \left( 4J_4^3 + \left( 2k_4 - 3i \left( \sqrt{X} + \sqrt{Y_2} \right) \right)^3 \right) \right)}{\sqrt{Y_2} \left( 4\sqrt{X}\sqrt{Y_2} + 4X - Y_1 + Y_2 \right)} \right\},
\end{aligned}$$

and the non-diagonal terms read

$$\begin{aligned}
A_{14} = & -4\lambda_1\lambda_2\sqrt{(n+1)(n+2)} \left\{ \frac{\left( \sqrt{Y_1} - \sqrt{X} \right) e^{-\frac{1}{2}t(\sqrt{X}-\sqrt{Y_1})}}{\sqrt{Y_1} \left( -4\sqrt{X}\sqrt{Y_1} + 4X + Y_1 - Y_2 \right)} + \frac{\left( \sqrt{X} + \sqrt{Y_1} \right) e^{-\frac{1}{2}t(\sqrt{X}+\sqrt{Y_1})}}{\sqrt{Y_1} \left( 4\sqrt{X}\sqrt{Y_1} + 4X + Y_1 - Y_2 \right)} \right. \\
& \left. + \frac{\left( \sqrt{X} + \sqrt{Y_2} \right) e^{\frac{1}{2}t(\sqrt{X}+\sqrt{Y_2})}}{\sqrt{Y_2} \left( 4\sqrt{X}\sqrt{Y_2} + 4X - Y_1 + Y_2 \right)} + \frac{\left( \sqrt{Y_2} - \sqrt{X} \right) e^{\frac{1}{2}t(\sqrt{X}-\sqrt{Y_2})}}{\sqrt{Y_2} \left( -4\sqrt{X}\sqrt{Y_2} + 4X - Y_1 + Y_2 \right)} \right\},
\end{aligned}$$

$$\begin{aligned}
A_{23} = & - \frac{2e^{\frac{1}{2}t(\sqrt{X}+\sqrt{Y_2})} \left( i(\Delta_1 + \Delta_2) \lambda_1\lambda_2 + (2n+3)\sqrt{X} + (2n+3)\sqrt{Y_2} \right)}{\sqrt{Y_2} \left( 4\sqrt{X}\sqrt{Y_2} + 4X - Y_1 + Y_2 \right)} \\
& + \frac{e^{-\frac{1}{2}t(\sqrt{X}+\sqrt{Y_1})} \left( 2i(\Delta_1 + \Delta_2) \lambda_1\lambda_2 - 2(2n+3)\sqrt{X} - 2(2n+3)\sqrt{Y_1} \right)}{\sqrt{Y_1} \left( 4\sqrt{X}\sqrt{Y_1} + 4X + Y_1 - Y_2 \right)}
\end{aligned}$$



$$\begin{aligned}
& + \frac{e^{\frac{1}{2}t(\sqrt{X}-\sqrt{Y_2})} \left( 2i(\Delta_1 + \Delta_2) \lambda_1 \lambda_2 + (4n+6)\sqrt{X} - 2(2n+3)\sqrt{Y_2} \right)}{\sqrt{Y_2} \left( -4\sqrt{X}\sqrt{Y_2} + 4X - Y_1 + Y_2 \right)} \\
& + \frac{e^{-\frac{1}{2}t(\sqrt{X}-\sqrt{Y_1})} \left( (4n+6)\sqrt{X} - 2((2n+3)\sqrt{Y_1} + i(\Delta_1 + \Delta_2) \lambda_1 \lambda_2) \right)}{\sqrt{Y_1} \left( -4\sqrt{X}\sqrt{Y_1} + 4X + Y_1 - Y_2 \right)},
\end{aligned}$$

$$\begin{aligned}
A_{13} = -i\lambda_1\sqrt{n+1} \left\{ & - \frac{e^{\frac{1}{2}t(\sqrt{X}-\sqrt{Y_2})} \left( G + (\sqrt{X} - \sqrt{Y_2}) \left( -2i\Delta_2 + \sqrt{X} - \sqrt{Y_2} \right) \right)}{\sqrt{Y_2} \left( -4\sqrt{X}\sqrt{Y_2} + 4X - Y_1 + Y_2 \right)} \right. \\
& + \frac{e^{\frac{1}{2}t(\sqrt{X}+\sqrt{Y_2})} \left( G + (\sqrt{X} + \sqrt{Y_2}) \left( -2i\Delta_2 + \sqrt{X} + \sqrt{Y_2} \right) \right)}{\sqrt{Y_2} \left( 4\sqrt{X}\sqrt{Y_2} + 4X - Y_1 + Y_2 \right)} \\
& + \frac{e^{-\frac{1}{2}t(\sqrt{X}-\sqrt{Y_1})} \left( G + (\sqrt{X} - \sqrt{Y_1}) \left( 2i\Delta_2 + \sqrt{X} - \sqrt{Y_1} \right) \right)}{\sqrt{Y_1} \left( -4\sqrt{X}\sqrt{Y_1} + 4X + Y_1 - Y_2 \right)} \\
& \left. + \frac{e^{-\frac{1}{2}t(\sqrt{X}+\sqrt{Y_1})} \left( -G - (\sqrt{X} + \sqrt{Y_1}) \left( 2i\Delta_2 + \sqrt{X} + \sqrt{Y_1} \right) \right)}{\sqrt{Y_1} \left( 4\sqrt{X}\sqrt{Y_1} + 4X + Y_1 - Y_2 \right)} \right\},
\end{aligned}$$

$$\begin{aligned}
A_{12} = -i\sqrt{n+1}\lambda_2 \left\{ & \frac{e^{\frac{1}{2}t(\sqrt{X}-\sqrt{Y_2})} \left( G - (\sqrt{X} - \sqrt{Y_2}) \left( -2i\Delta_1 + \sqrt{X} - \sqrt{Y_2} \right) \right)}{\sqrt{Y_2} \left( -4\sqrt{X}\sqrt{Y_2} + 4X - Y_1 + Y_2 \right)} \right. \\
& + \frac{e^{\frac{1}{2}t(\sqrt{X}+\sqrt{Y_2})} \left( -G + (\sqrt{X} + \sqrt{Y_2}) \left( -2i\Delta_1 + \sqrt{X} + \sqrt{Y_2} \right) \right)}{\sqrt{Y_2} \left( 4\sqrt{X}\sqrt{Y_2} + 4X - Y_1 + Y_2 \right)} \\
& + \frac{e^{-\frac{1}{2}t(\sqrt{X}-\sqrt{Y_1})} \left( -G + (\sqrt{X} - \sqrt{Y_1}) \left( 2i\Delta_1 + \sqrt{X} - \sqrt{Y_1} \right) \right)}{\sqrt{Y_1} \left( -4\sqrt{X}\sqrt{Y_1} + 4X + Y_1 - Y_2 \right)} \\
& \left. + \frac{e^{-\frac{1}{2}t(\sqrt{X}+\sqrt{Y_1})} \left( G - (\sqrt{X} + \sqrt{Y_1}) \left( 2i\Delta_1 + \sqrt{X} + \sqrt{Y_1} \right) \right)}{\sqrt{Y_1} \left( 4\sqrt{X}\sqrt{Y_1} + 4X + Y_1 - Y_2 \right)} \right\},
\end{aligned}$$

$$\begin{aligned}
A_{24} = -i\sqrt{n+2}\lambda_1 \left\{ & \frac{e^{-\frac{1}{2}t(\sqrt{X}-\sqrt{Y_1})} \left( H + (\sqrt{X} - \sqrt{Y_1}) \left( -2i\Delta_2 + \sqrt{X} - \sqrt{Y_1} \right) \right)}{\sqrt{Y_1} \left( -4\sqrt{X}\sqrt{Y_1} + 4X + Y_1 - Y_2 \right)} \right. \\
& + \frac{e^{-\frac{1}{2}t(\sqrt{X}+\sqrt{Y_1})} \left( -H - (\sqrt{X} + \sqrt{Y_1}) \left( -2i\Delta_2 + \sqrt{X} + \sqrt{Y_1} \right) \right)}{\sqrt{Y_1} \left( 4\sqrt{X}\sqrt{Y_1} + 4X + Y_1 - Y_2 \right)} \\
& \left. + \frac{e^{\frac{1}{2}t(\sqrt{X}+\sqrt{Y_2})} \left( H + (\sqrt{X} + \sqrt{Y_2}) \left( 2i\Delta_2 + \sqrt{X} + \sqrt{Y_2} \right) \right)}{\sqrt{Y_2} \left( 4\sqrt{X}\sqrt{Y_2} + 4X - Y_1 + Y_2 \right)} \right\}
\end{aligned}$$

$$\begin{aligned}
& - \frac{e^{\frac{1}{2}t(\sqrt{X}-\sqrt{Y_2})} \left( H + \left( \sqrt{X} - \sqrt{Y_2} \right) \left( 2i\Delta_2 + \sqrt{X} - \sqrt{Y_2} \right) \right)}{\sqrt{Y_2} \left( -4\sqrt{X}\sqrt{Y_2} + 4X - Y_1 + Y_2 \right)} \Bigg\}, \\
A_{34} = & -i\sqrt{n+2}\lambda_2 \left\{ \frac{e^{-\frac{1}{2}t(\sqrt{X}-\sqrt{Y_1})} \left( -H + \left( \sqrt{X} - \sqrt{Y_1} \right) \left( -2i\Delta_1 + \sqrt{X} - \sqrt{Y_1} \right) \right)}{\sqrt{Y_1} \left( -4\sqrt{X}\sqrt{Y_1} + 4X + Y_1 - Y_2 \right)} \right. \\
& + \frac{e^{-\frac{1}{2}t(\sqrt{X}+\sqrt{Y_1})} \left( H - \left( \sqrt{X} + \sqrt{Y_1} \right) \left( -2i\Delta_1 + \sqrt{X} + \sqrt{Y_1} \right) \right)}{\sqrt{Y_1} \left( 4\sqrt{X}\sqrt{Y_1} + 4X + Y_1 - Y_2 \right)} \\
& + \frac{e^{\frac{1}{2}t(\sqrt{X}-\sqrt{Y_2})} \left( H - \left( \sqrt{X} - \sqrt{Y_2} \right) \left( 2i\Delta_1 + \sqrt{X} - \sqrt{Y_2} \right) \right)}{\sqrt{Y_2} \left( -4\sqrt{X}\sqrt{Y_2} + 4X - Y_1 + Y_2 \right)} \\
& \left. + \frac{e^{\frac{1}{2}t(\sqrt{X}+\sqrt{Y_2})} \left( -H + \left( \sqrt{X} + \sqrt{Y_2} \right) \left( 2i\Delta_1 + \sqrt{X} + \sqrt{Y_2} \right) \right)}{\sqrt{Y_2} \left( 4\sqrt{X}\sqrt{Y_2} + 4X - Y_1 + Y_2 \right)} \right\}.
\end{aligned}$$

In the expressions above, the coefficients  $X$  and  $Y_{1,2}$  are given by

$$X = -\frac{2A}{3} + \frac{2^{1/3} (A^2 + 12F)}{3\beta} + \frac{\Gamma}{3 \times 2^{1/3}}$$

$$Y_{1,2} = -2A - X \pm \frac{2iB}{\sqrt{X}},$$

where

$$\Gamma = \left[ 2A^3 - 27B^2 - 72AF + \sqrt{-4(A^2 + 12F)^3 + (2A^3 - 27B^2 - 72AF)^2} \right]^{1/3},$$

and

$$A = \frac{1}{2} (\Delta_1^2 + \Delta_2^2 + 2(2n+3)(\lambda_1^2 + \lambda_2^2)),$$

$$B = (\Delta_1\lambda_2^2 + \Delta_2\lambda_1^2),$$

$$F = \frac{1}{16} \left\{ (\Delta_1^2 - \Delta_2^2) \left[ (\Delta_1^2 - \Delta_2^2)^2 + 4(3+2n)(\lambda_1^2 - \lambda_2^2) \right] + 16(n^2 + 3n + 2)(\lambda_1^2 - \lambda_2^2)^2 \right\}.$$

We have also that

$$G = \Delta_1^2 - \Delta_2^2 + 4(n+2)(\lambda_1^2 - \lambda_2^2),$$

$$H = \Delta_1^2 - \Delta_2^2 + 4(n+1)(\lambda_1^2 - \lambda_2^2),$$

$$D_i = \frac{(-1)^{5/6} (3a_i + k_i^2)}{3J_i}, \quad i = 1, 2, 3, 4,$$

and

$$J_i = \left( -27b_i + 9a_i k_i + 2k_i^3 - 3i\sqrt{3}\sqrt{4a_i^3 - 27b_i^2 + 18a_i b_i k_i + a_i^2 k_i^2 + 4b_i k_i^3} \right)^{1/3}, \quad i = 1, 2, 3, 4,$$

being

$$a_1 = a_4 = \frac{[(\Delta_1 - \Delta_2)^2 + 4(n+2)(\lambda_1^2 + \lambda_2^2)]}{4},$$

$$a_2 = a_3 = \frac{[(\Delta_1 + \Delta_2)^2 + 4((n+1)\lambda_1^2 + (n+2)\lambda_2^2)]}{4},$$

$$b_1 = -b_4 = -\frac{(\Delta_1 - \Delta_2)}{8} [\Delta_1^2 - \Delta_2^2 + 4(n+2)(\lambda_1^2 - \lambda_2^2)],$$

$$b_2 = -b_3 = -\frac{(\Delta_1 + \Delta_2)}{8} [\Delta_1^2 - \Delta_2^2 + 4((n+1)\lambda_1^2 - (n+2)\lambda_2^2)],$$

$$k_1 = -k_4 = -\frac{(\Delta_1 + \Delta_2)}{2},$$

and

$$k_2 = -k_3 = -\frac{(\Delta_1 - \Delta_2)}{2}.$$

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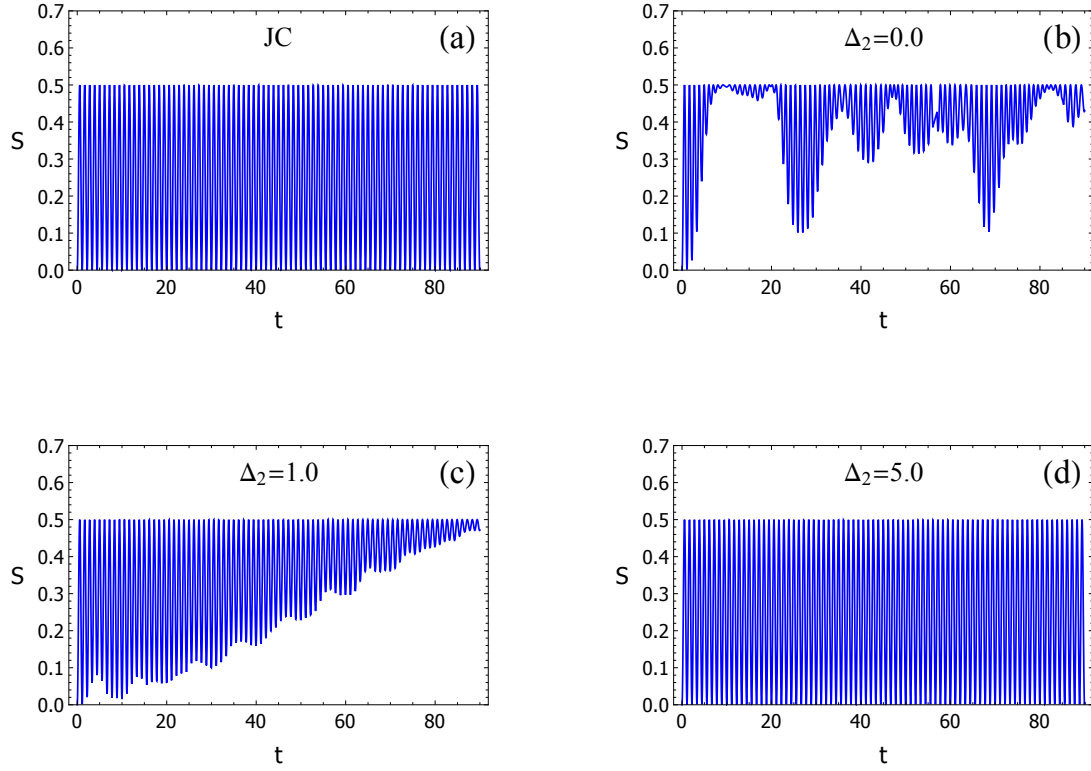


Figure 1: Linear entropy of atom 1 as a function of time. In (a) the field is not coupled to the environment, i.e.,  $\lambda_2 = 0.0$  (Jaynes Cummings model). For the field coupled to the environment, with  $\lambda_2 = 0.1$  and (b)  $\Delta_2 = 0.0$ ; (c)  $\Delta_2 = 1.0$ , and (d)  $\Delta_2 = 5.0$ . In all cases  $\lambda_1 = 1.0$ , and the initial state of the system is a tripartite product state with  $|\psi_{a1}\rangle = |e_1\rangle$ ,  $\rho_{a2} = \frac{1}{2}(|g_2\rangle\langle g_2| + |e_2\rangle\langle e_2|)$  and  $|\psi_f\rangle = |1\rangle$ .

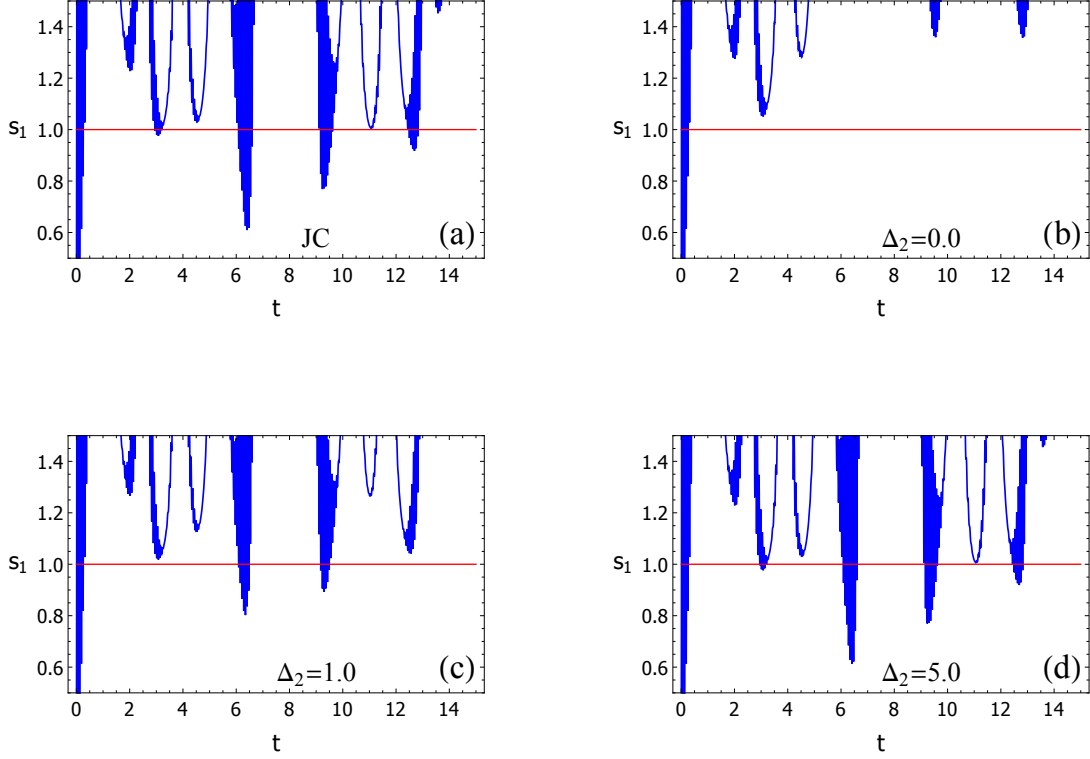


Figure 2: Squeezing index  $s_1$  as a function of time. In (a) the field is not coupled to the environment, i.e.,  $\lambda_2 = 0.0$  (Jaynes Cummings model). For the field coupled to the environment, with  $\lambda_2 = 0.1$  and (b)  $\Delta_2 = 0.0$ ; (c)  $\Delta_2 = 1.0$ , and (d)  $\Delta_2 = 5.0$ . In all cases  $\lambda_1 = 1.0$ , and the initial state of the system is a tripartite product state with  $|\psi_{a1}\rangle = \cos(0.6)|g_1\rangle + \sin(0.6)|e_1\rangle$ ,  $\rho_{a2} = \frac{1}{2}(|g_2\rangle\langle g_2| + |e_2\rangle\langle e_2|)$  and  $|\psi_f\rangle = |1\rangle$ .

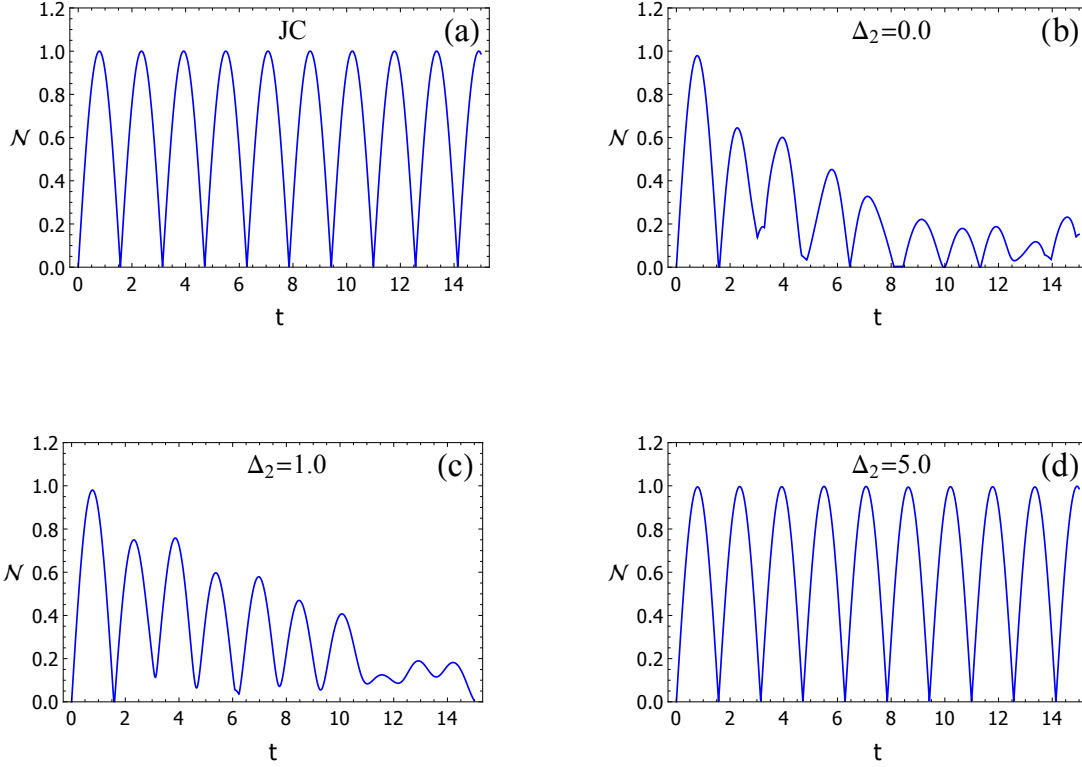


Figure 3: Negativity  $\mathcal{N}$  used to quantify the entanglement between atom 1 and the field as a function of time. In (a) the field is not coupled to the environment, i.e.,  $\lambda_2 = 0.0$  (Jaynes Cummings model). For the field coupled to the environment, with  $\lambda_2 = 0.2$  and (b)  $\Delta_2 = 0.0$ ; (c)  $\Delta_2 = 1.0$ , and (d)  $\Delta_2 = 5.0$ . In all cases  $\lambda_1 = 1.0$ , and the initial state of the system is a tripartite product state with  $|\psi_{a1}\rangle = |e_1\rangle$ ,  $\rho_{a2} = \frac{1}{2}(|g_2\rangle\langle g_2| + |e_2\rangle\langle e_2|)$  and  $|\psi_f\rangle = |0\rangle$ .

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